Oct. 2010 CHINESE JOURNAL OF ENGINEERING MATHEMATICS Vol. 27 No. 5

Article ID:1005-3085(2010)05-0932-07

A Pinching Theorem for Totally Real Riemannian Foliations with Parallelized Mean Curvature Vectors on a Quaternion Projective Space

PENG Hui-chun[1], LI Zhi-bo[2]

(1- Department of Mathematics and Physics, North China Electric Power University, Beijing 102206;
2- Department of Mathematics, Zhengzhou University, Zhengzhou 450052)

Abstract: By utilizing the divergence evaluation method of Nakagawa and Takagi for Harmonic foliations on the sphere, the divergence of a vector field on a totally real Riemannian foliation with parallelized mean curvature vectors on a quaternion projective space is found out. By pinching the length of the second fundamental form, a formula of Simons' type is obtained. The geometric restrictions of the length of the second fundamental form are proposed so that each leave of foliation is assumed to be totally umbilical.

Keywords: Riemannian foliations; quaternion projective space; mean curvature; divergence; totally umbilical

Classification: AMS(2000) 53C42 **CLC number:** O186 **Document code:** A

1 Introduction

Geometric notions in the theory of Riemannian submanifolds have their counterparts for foliations on Riemannian manifolds. The harmonic foliations on Riemannian manifolds have been extensively studied in recent years[1-4]. Many harmonic foliations which are not totally geodesic are known. It has been known that under some geometric restrictions, harmonicity implies totally geodesicness. But the geometric property of Riemannian foliations with parallelized mean curvature vectors in spaces are still unknown. The purpose of this paper is to study the totally real Riemannian foliations with parallelized mean curvature vectors on a quaternion projective space. Improving the method of Nakagawa and Takagi[3], we calculate divergence of the vector field and obtained a formula of Simons'. Then We prove that the totally real Riemannian foliations with parallelized mean curvature vectors on a quaternion projective space satisfying the length square of second fundamental form

$$S \le \min\left\{\frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3}\right\}, \quad c>0, \quad n>2,$$

Received: 07 July 2008. **Accepted:** 22 Feb 2010.

Biography: Peng Huichun (Born in 1971), Female, Master, Associate Professor. Research field: differential geometry.

is totally umbilical, or

$$S = \min\left\{\frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3}\right\}.$$

2 Preliminaries

Let $QP^n(c)$ be a $4n$-dimensional ($n > 2$) quaternion projective space with constant quaternion sectional curvature $c > 0$. A submanifold M is said to be a totally real submanifold of $QP^n(c)$ if every tangent space of M is mapped into its totally real normal space by the quaternion structures of $QP^n(c)$. $\mathbf{F}$ is said to be totally real foliation in $QP^n(c)$, if each leaf $\mathbf{L}$ of $\mathbf{F}$ is a totally real submanifold in $QP^n(c)$. Let $\mathbf{F}$ be a totally real n-dimensional foliation on $QP^n(c)$. Considering $\mathbf{F}$ as a integrable distribution on $QP^n(c)$, we denote the orthogonal distribution of $\mathbf{F}$ by $\mathbf{F}^{\perp}$, which is called the normal plane field. For any vector field X on $QP^n(c)$, we decompose it as $X = X' + X''$, where X' (resp. X'') is tangent (resp. normal) to $\mathbf{F}$. We define two tensors A and h of type (1,2) on $QP^n(c)$ by

$$A(X,Y) = -(\nabla_{Y''}X'')', \quad h(X,Y) = -(\nabla_{Y'}X')'', \tag{1}$$

for any vector fields X and Y on $QP^n(c)$, where ∇ denotes the Riemannian connection with respect to the Riemannian metric g of $QP^n(c)$.

The restriction of h to each leaf of $\mathbf{F}$ is called the second fundamental form of the leaf. We define the second fundamental form B of the normal field $\mathbf{F}^{\perp}$ by[2]

$$B(X,Y) = \frac{1}{2}\{A(X,Y) + A(Y,X)\}, \tag{2}$$

for any vector fields X and Y on $QP^n(c)$. We will use throughout this paper, the following convention on the range of indices unless otherwise stated:

$$A, B, C, \cdots = 1, \cdots, n, \quad I(1), \cdots, I(n), \quad J(1), \cdots, J(n),$$
$$K(1), \cdots, K(n); \quad i, j, k, \cdots = 1, 2, \cdots, n;$$
$$\alpha, \beta, \gamma, \cdots = I(1), \cdots, I(n), \quad J(1), \cdots, J(n), \quad K(1), \cdots, K(n).$$

Let $e_1, \cdots, e_n$, $e_{I(1)}, \cdots, e_{I(n)}$, $e_{J(1)}, \cdots, e_{J(n)}$, $e_{K(1)}, \cdots, e_{K(n)}$ be a locally defined orthonormal frame field of $QP^n(c)$ such that, restricting to $\mathbf{F}$, $e_1, \cdots, e_n$ are tangent to $\mathbf{F}$. Let $\{\omega_A\}$ be the dual frame field. The structure equations of $QP^n(c)$ are given as follows[5]:

$$d\omega_A = -\omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0, \tag{3}$$

$$d\omega_{AB} = -\omega_{AC} \wedge \omega_{CB} + \frac{1}{2} K_{ABCD}\omega_C \wedge \omega_D, \tag{4}$$

$$\begin{aligned} K_{ABCD} = \frac{C}{4}(&\delta_{AC}\delta_{BD} - \delta_{AD}\delta_{BC} + I_{AC}I_{BD} - I_{AD}I_{BC} + 2I_{AB}I_{CD} + J_{AC}J_{BD} \\ &- J_{AD}J_{BC} + 2J_{AB}J_{CD} + K_{AC}K_{BD} - K_{AD}K_{BC} + 2K_{AB}J_{CD}), \end{aligned} \tag{5}$$

where I, J and K are quaternion structures of $QP^n(c)$, and K_{ABCD} is the curvature tensor of $QP^n(c)$. Restrict to $\mathbf{F}$:

$$\omega_\alpha = 0, \quad \omega_{ij} = \omega_{\alpha\beta}, \quad \omega_{\alpha i} = \omega_{i\alpha}, \tag{6}$$

$$\omega_{\alpha i} = \sum h^\alpha_{ij}\omega_j + \sum A^j_{\alpha\beta}\omega_\beta, \quad h^\alpha_{ij} = h^\alpha_{ji} = h^\beta_{jk} = h^\beta_{ik}, \tag{7}$$

$$d\omega_i = -\omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0, \tag{8}$$

$$d\omega_{ij} = -\omega_{ik} \wedge \omega_{kj} + \frac{1}{2}R_{ijkl}\omega_k \wedge \omega_l, \tag{9}$$

$$d\omega_{\alpha\beta} = -\omega_{\alpha\gamma} \wedge \omega_{\gamma\beta} + \frac{1}{2}R_{\alpha\beta kl}\omega_k \wedge \omega_l, \tag{10}$$

where R_{ijkl} and $R_{\alpha\beta kl}$ denote the curvature tensor of tangent connection and normal connection of $\mathbf{F}$, respectively. The Riemannian connection ∇ on $QP^n(c)$ is given by $\nabla_{e_A} e_B = \sum \omega_{CB}(e_A)e_C$. Then the components h^A_{BC} (resp. A^B_{CD}) of h (resp. A) with respect to $\{e_A\}$ and $\{\omega_A\}$ are given by

$$h^\alpha_{ij} = \omega_{\alpha i}(e_j), \quad A^i_{\alpha\beta} = \omega_{\alpha i}(e_\beta). \tag{11}$$

The second fundamental form $\mathbf{H}$ of $\mathbf{F}$ is: $\mathbf{H} = \sum_{a,i,j} h^\alpha_{ij}\omega_i\omega_j e_\alpha$. The length square of $\mathbf{H}$ is: $S = \sum_{a,i,j}(h^\alpha_{ij})^2$. For each α, H^α denotes the matrix (h^α_{ij}). $\xi = \frac{1}{n}\sum_\alpha(\mathrm{tr}H^\alpha)e_\alpha$ is called the mean curvature vector, and $H = \|\xi\|$ is called the mean curvature, where tr denotes the trace of the matrix (h^α_{ij}). The foliation $\mathbf{F}$ is said to be harmonic or minimal (resp. totally geodesic) if $\sum h^\alpha_{jj} = 0$ (resp. $h^\alpha_{jj} = 0$). The normal plane field $\mathbf{F}^\perp$ is said to be minimal if $\mathrm{tr}B = \sum A^i_{\alpha\alpha}e_i = 0$. The normal plane field $\mathbf{F}^\perp$ is said to be totally geodesic, if $B = 0$. The metric is bundle-like if and only if $A^i_{\alpha\beta} = -A^i_{\beta\alpha}$, which imply $B = 0$. The foliations $\mathbf{F}$ with bundle-like metric is called Riemannian foliations. By (5), we have

$$R_{ijkl} = K_{ijkl} + \sum_\alpha(h^\alpha_{ik}h^\alpha_{jl} - h^\alpha_{il}h^\alpha_{jk}), \quad R_{\alpha\beta kl} = K_{\alpha\beta kl} + \sum_m(h^\alpha_{km}h^\beta_{ml} - h^\alpha_{km}h^\beta_{lm}). \tag{12}$$

For a tensor field $T = (T^{A_1\cdots A_r}_{B_1\cdots B_s})$ on $QP^n(c)$, we define its 1-order covariant derivatives by[9]

$$\begin{aligned} T^{A_1\cdots A_r}_{B_1\cdots B_s C}\omega_C &= dT^{A_1\cdots A_r}_{B_1\cdots B_s} - \sum T^{A_1\cdots A_{a-1},C,A_{a+1}\cdots A_r}_{B_1\cdots B_s}\omega_{CA_a} \\ &\quad - \sum T^{A_1\cdots A_r}_{B_1\cdots B_{b-1},C,B_{b+1}\cdots B_s}\omega_{CB_b}. \end{aligned} \tag{13}$$

Then we have the definiens of (h^A_{BCD}) and (A^A_{BCD}). The detailed formulae can be found in reference [2].

3 Calculus of the divergence

A vector field $v = \sum \nu_A e_A$ on $QP^n(c)$ is defined by

$$\nu_k = \sum h^\alpha_{ij}h^\alpha_{ijk}, \quad \nu_\alpha = 0. \tag{14}$$

By [6], we know that the divergence of the vector field v is defined by

$$\delta v = \mathrm{div}v = \sum \nu_{AA} = \sum \nu_{kk} + \sum \nu_{\alpha\alpha}. \tag{15}$$

Since $\mathbf{F}$ is a Riemannian foliations, i.e., $A^i_{\alpha\beta} = -A^i_{\beta\alpha}$, then

$$A^i_{\alpha\alpha} = 0. \tag{16}$$

Taking exterior differentiation of (14) and using (16), we have

$$\begin{aligned}\nu_{kk} &= \sum h^\alpha_{ijk} h^\alpha_{ijk} + \sum h^\alpha_{ij} h^\alpha_{ijkk} + \sum h^\alpha_{ij} h^\beta_{ij} h^\beta_{mk} h^\alpha_{mk} \\ &\quad + \sum h^\alpha_{ij} h^\alpha_{jl} h^\beta_{lk} h^\beta_{kj} + \sum h^\alpha_{ij} h^\alpha_{jl} h^\beta_{lk} h^\beta_{ki} + \sum h^\alpha_{ij} h^\alpha_{ij\beta} h^\beta_{kk}, \qquad (17)\\ \nu_{\alpha\alpha} &= \sum_{k,\alpha} \nu_k A^k_{\alpha\alpha} = 0.\end{aligned}$$

It have been obtained that[2,6]

$$\begin{aligned}h^\alpha_{ij} h^\alpha_{ijkk} \;=\; & \sum h^\alpha_{ij} h^l_{kkij} - 2\sum h^\alpha_{ij} h^\beta_{ij} h^\alpha_{lk} h^\beta_{lk} + 4\sum h^\alpha_{ij} h^\beta_{jl} h^\alpha_{lk} h^\beta_{ki} \\ & -2\sum h^\alpha_{ij} h^\alpha_{jl} h^\beta_{lk} h^\beta_{ki} - 2\sum h^\alpha_{ij} h^\beta_{jk} h^\beta_{kl} h^\alpha_{li} + 2\sum h^\alpha_{ij} h^\beta_{jl} h^\alpha_{li} h^\beta_{kk} \\ & -\sum h^\alpha_{ij} h^\alpha_{ij\beta} h^\alpha_{kk} + \sum h^\alpha_{ij} h^\alpha_{kk\beta} h^\beta_{ij} - c\sum_\alpha \Big(\sum_k h^\alpha_{kk}\Big)^2 + cn\sum_{\alpha,i,j} (h^\alpha_{ij})^2. \qquad (18)\end{aligned}$$

Now assume e_α to be the mean curvature vector. Hence $\sum_k h^\beta_{kk} = 0\,(\beta \neq \alpha)$. From (13), we have

$$\begin{cases} \sum_{k,c} h^\alpha_{kkc}\omega_c = ndH, \\ \sum_{k,c} h^\beta_{kkc}\omega_c = nH\omega_{\beta\alpha}, \quad \beta \neq \alpha. \end{cases} \tag{19}$$

Here H is the mean curvature. The vector e_α is parallel in the normal bundle.

According to (19), we have

$$\begin{aligned}h^\alpha_{ij} h^\alpha_{ijkk} \;=\; & -2\sum h^\alpha_{ij} h^\beta_{ij} h^\alpha_{lk} h^\beta_{lk} + 2\sum h^\alpha_{ij} h^\beta_{jl} h^\alpha_{lk} h^\beta_{ki} - 2\sum h^\alpha_{ij} h^\alpha_{jl} h^\beta_{lk} h^\beta_{ki} \\ & -2\sum h^\alpha_{ij} h^\beta_{jk} h^\beta_{kl} h^\alpha_{li} + \sum h^\alpha_{ij} h^\beta_{jl} h^\alpha_{li} h^\beta_{kk} - \sum h^\alpha_{ij} h^\alpha_{ij\beta} h^\beta_{kk} \\ & -c\sum_\alpha \Big(\sum_k h^\alpha_{kk}\Big)^2 + cn\sum_{\alpha,i,j} (h^\alpha_{ij})^2. \qquad (20)\end{aligned}$$

Let H^α denote the matrix (h^α_{ij}). The divergence of the vector field v is simplified by

$$\begin{aligned}\delta v \;=\; & \sum h^\alpha_{ijk} h^\alpha_{ijk} + \sum_{\alpha,\beta} \mathrm{tr}(H^\alpha H^\beta - H^\beta H^\alpha)(H^\alpha H^\beta - H^\beta H^\alpha) \\ & -\sum_{\alpha,\beta} \left[\mathrm{tr}(H^\alpha H^\beta)\right]^2 + \sum_{\alpha,\beta} \left[\mathrm{tr}(H^\alpha)^2 H^\beta\right]\mathrm{tr}(H^\beta) \\ & -c\sum_\alpha \left(\mathrm{tr}(H^\alpha)\right)^2 + cn\sum_\alpha \mathrm{tr}(H^\alpha)^2. \qquad (21)\end{aligned}$$

4 Proof of the main theorems

Lemma 4.1[6] Let A, B be an $n\times n$ symmetric matrix and $N(A)=\mathrm{tr}(AA^T)$. Then

$$N(AB-BA)\le 2N(A)N(B).$$

Lemma 4.2[7] There are $2n$ real numbers $a_1,a_2,\cdots,a_n$ and $b_1,b_2,\cdots,b_n$ with $\sum_{i=1}^n b_i=0$. Then

$$\Big(\sum_{i,j}a_ia_j(b_i-b_j)^2\Big)^2\le(2n+6)\Big(\sum a_i^2\Big)^2\Big(\sum b_j^2\Big)^2.$$

Theorem 4.1 Let $QP^n(c)$ be a $4n$−dimensional $(n>2)$ quaternion projective space with constant quaternion sectional curvature $c>0$, and $\mathbf{F}$ be a totally real Riemannian foliation in $QP^n(c)$ with parallelized mean curvature vectors, and the mean curvature be $H(\neq 0)$ with $n\ge 2$. Let e_γ be the mean curvature vector. Hence

$$\int_{QP^n(c)}\sum_{\alpha\neq\gamma}(h_{ij}^\alpha)^2\Big\{\max\Big(\sqrt{\frac{n+3}{2}},\frac{2n-3}{n-1}\Big)S-cn\Big\}*1\ge 0.$$

Proof Assume that $\mathbf{F}$ is a foliation with parallelized mean curvature vectors and the mean curvature is $H(\neq 0)$. Let $e_\gamma=\xi/\|\xi\|$. Then we have

$$\begin{cases}\mathrm{tr}(H^\alpha)=0, & \alpha\neq\gamma,\\ \mathrm{tr}(H^\gamma)=nH.\end{cases}\tag{22}$$

From $\omega_{\gamma,\alpha}=0$, we have

$$H^\gamma H^\alpha=H^\alpha H^\gamma.\tag{23}$$

From (22) and (23), (21) becomes

$$\delta v_\gamma=h_{ijk}^\gamma h_{ijk}^\gamma-\sum_{\beta\neq\gamma}\big[\mathrm{tr}(H^\gamma H^\beta)\big]^2-cn^2H^2+cn\mathrm{tr}(H^\gamma)^2,\tag{24}$$

$$\begin{aligned}-\delta v_{(\alpha)}+\sum_{\alpha\neq\gamma}h_{ijk}^\alpha h_{ijk}^\alpha=&-\sum_{\alpha,\beta\neq\gamma}\mathrm{tr}(H^\alpha H^\beta-H^\beta H^\alpha)^2+\sum_{\alpha,\beta\neq\gamma}\big[\mathrm{tr}(H^\alpha H^\beta)\big]^2\\&+\sum_{\alpha\neq\gamma}\big[\mathrm{tr}(H^\alpha H^\gamma)\big]^2-\sum_{\alpha\neq\gamma}nH\mathrm{tr}\big[(H^\alpha)^2H^\gamma\big]-cn\sum_{\alpha\neq\gamma}(h_{ij}^\alpha)^2.\end{aligned}\tag{25}$$

Let

$$I=-\sum_{\alpha,\beta\neq\gamma}\mathrm{tr}(H^\alpha H^\beta-H^\beta H^\alpha)^2+\sum_{\alpha,\beta\neq\gamma}\big[\mathrm{tr}(H^\alpha H^\beta)\big]^2,\tag{26}$$

$$II=\sum_{\alpha\neq\gamma}\big[\mathrm{tr}(H^\alpha H^\gamma)\big]^2-\sum_{\alpha\neq\gamma}nH\mathrm{tr}\big[(H^\alpha)^2H^\gamma\big].\tag{27}$$

From (26) and Lemma 4.1, we have

$$\begin{aligned}I&=(2(n-1)^2-(n-1))\sigma_1^2-(n-1)^2(n-2)(\sigma_1^2-\sigma_2)\\&\le(n-1)(2n-3)\sigma_1^2\le\Big(\frac{2n-3}{n-1}\Big)\Big(\sum_{\alpha,\beta\neq\gamma}S_\alpha\Big)^2,\end{aligned}$$

where

$$(n-1)\sigma_1 = \sum_{\alpha,\beta\neq\gamma} S_\alpha, \quad \frac{(n-1)(n-2)}{2}\sigma_2 = \sum_{\alpha\neq\beta;\alpha,\beta\neq\gamma} S_\alpha S_\beta.$$

From (27) and for a fixed α, we choose a local field of orthonormal frames $\{e_1,\cdots,e_n\}$ such that the matrix (h^α_{ij}) is diagonal, i.e., $h^\alpha_{ij}=0\,(i\neq j)$. By applying (22) and Lemma 4.2, we obtain

$$\begin{aligned} II &= \Big(\sum_i h^\gamma_{ii}h^\alpha_{ii}\Big)^2 - \sum_{i,j} h^\gamma_{ii}h^\gamma_{jj}(h^\alpha_{jj})^2 \\ &= \sum_{i,j}\,[h^\gamma_{ii}h^\alpha_{ii}h^\gamma_{jj}h^\alpha_{jj} - h^\gamma_{ii}h^\gamma_{jj}(h^\alpha_{jj})^2] = -\frac{1}{2}\sum_{i,j} h^\gamma_{ii}h^\gamma_{jj}(h^\alpha_{ii}-h^\alpha_{jj})^2 \\ &\leq \frac{\sqrt{2n+1}}{2}\Big(\sum_i h^\gamma_{ii}\Big)^2\Big(\sum_j (h^\alpha_{ii})^2\Big) = \sqrt{\frac{n+3}{2}}\Big(\sum_{i,j}(h^\gamma_{ij})^2\Big)\Big(\sum_{\alpha\neq\gamma}(h^\alpha_{ij})^2\Big). \end{aligned}$$

Then

$$\begin{aligned} -\delta v_{\alpha\neq\gamma} + \sum_{\alpha\neq\gamma} h^\alpha_{ijk}h^\alpha_{ijk} &\leq \sum_{\alpha\neq\gamma}(h^\alpha_{ij})^2\Big\{\sqrt{\frac{n+3}{2}}\sum_{i,j}(h^\gamma_{ij})^2 + \frac{2n-3}{n-1}\sum_{\alpha\neq\gamma}(h^\alpha_{ij})^2 - cn\Big\} \\ &\leq \sum_{\alpha\neq\gamma}(h^\alpha_{ij})^2\Big\{\max\Big(\sqrt{\frac{n+3}{2}}, \frac{2n-3}{n-1}\Big)S - cn\Big\}. \end{aligned}$$

Taking integration on both sides of the above inequality and according to the Stokes theorem, we have

$$\begin{aligned} 0 &\leq \int_{QP^n(c)} \sum_{\alpha\neq\gamma} h^\alpha_{ijk}h^\alpha_{ijk} * 1 \\ &\leq \int_{QP^n(c)} \sum_{\alpha\neq\gamma}(h^\alpha_{ij})^2\Big\{\max\Big(\sqrt{\frac{n+3}{2}}, \frac{2n-3}{n-1}\Big)S - cn\Big\} * 1. \end{aligned} \tag{28}$$

The theorem is proved.

Corollary 4.1 Let **F** be a totally real Riemannian foliation in $QP^n(c)$ with parallelized mean curvature vectors and the mean curvature be $H(\neq 0)$ with $n\geq 2$. Let e_γ be the mean curvature vector. If the length square of second fundamental form

$$S \leq \min\Big\{\frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3}\Big\},$$

then each leaf of **F** is totally umbilical, or

$$S = \min\Big\{\frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3}\Big\}.$$

Proof Since

$$S \leq \min\Big\{\frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3}\Big\},$$

we have

$$\left\{\max\left(\sqrt{\frac{n+3}{2}}, \frac{2n-3}{n-1}\right)S - cn\right\} \leq 0.$$

From (28), we obtain

$$S = \min\left\{\frac{cn}{\sqrt{\frac{n+3}{2}}}, \frac{cn(n-1)}{2n-3}\right\},$$

or

$$\sum_{\alpha\neq\gamma}(h_{ij}^{\alpha})^2 = 0. \tag{29}$$

Let $\mu_{ij} = (h_{ij}^{\gamma} - H\delta_{ij})$ and $U = (\mu_{ij})$. Then $\mathrm{tr}(U^2) = \sum_{i,j}^{n}(\mu_{ij})^2 = \mathrm{tr}(H^{\gamma})^2 - nH^2$. From (24) and (29), we have

$$-\delta v_{(\gamma)} + h_{ijk}^{\gamma}h_{ijk}^{\gamma} \leq \mathrm{tr}(H^{\gamma})^2\sum_{\beta\neq\gamma}\mathrm{tr}(H^{\beta})^2 - cn\mathrm{tr}U^2 = -cn\mathrm{tr}U^2.$$

Taking integration on both sides of the above inequality and using the Stokes theorem, we have

$$0 \leq \int_{QP^n(c)} h_{ijk}^{\gamma}h_{ijk}^{\gamma} * 1 \leq \int_{QP^n(c)}(-cn\mathrm{tr}U^2) * 1.$$

It implies $\mathrm{tr}(U^2) = 0$, i.e., $\mu_{ij} = (h_{ij}^{\gamma} - H\delta_{ij}) = 0$. The corollary obviously holds. When $n < 5$, the corollary implies Theorem 1 in [4].

References:

[1] Gotoh T. Harmonic foliations on a complex projective space[J]. Tsukuba Journal of Mathematics, 1991, 15: 397-407

[2] Li Z B. Harmonic foliations on the sphere[J]. Tsukuba Journal of Mathematics, 1991, 15: 397-407

[3] Nakagawa H, Takagi R. Harmonic foliations on a compact Riemannian manifold of non-negative constant curvature[J]. Tohoku Journal of Mathematics, 1988, 40: 465-471

[4] Shu S C. Totally real harmonic foliations on a quaternion projective space[J]. Chinese Journal of Engineering Mathematics, 1999, 16(2): 19-24

[5] Chen B Y, Ogiue K. On totally real submanifolds[J]. Transactions of American Mathematical Society, 1974, 193: 257-266

[6] Li A M, Li J M. An intrinsic rigidity theorem for minimal submanifolds in a sphere[J]. Arch Mathematics, 1992, 58: 582-594

[7] Chen Q M. Maximun principle in the geometry of submanifolds[J]. Mathematics Zeit, Springer Verlag, 1991, 206: 333-339

四元数射影空间中具有平行平均曲率向量的全实叶的Pinching定理

彭慧春[1]， 李志波[2]

(1- 华北电力大学数理系，北京 102206; 2- 郑州大学数学系，郑州 450052)

摘 要: 本文利用Nakagawa和Takagi的计算散度的方法，求出四元数射影空间上具有平行平均曲率向量的全实黎曼叶状结构**F**的向量场的散度，并证明了其上的一个Simons型的Pinching定理。

关键词: 黎曼叶状结构；四元数射影空间；平均曲率；散度；全脐